Physics Letters B 545 (2002) 352–360

PHYSICS LETTERS B

www.elsevier.com/locate/npe

A predictive ansatz for neutrino mixing and leptogenesis

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Received 30 July 2002; received in revised form 26 August 2002; accepted 27 August 2002

Editor: T. Yanagida

Abstract

We propose a non-SO(10) modification of the Buchmüller–Wyler ansatz for neutrino mixing and leptogenesis, in which charged lepton, Dirac and Majorana neutrino mass matrices have fewer free parameters. Predictions of this new ansatz for three light neutrino masses, three lepton flavor mixing angles, the neutrinoless double-β decay and the cosmological matter–anti-matter asymmetry are all in very good agreement with current experimental and observational data.

PACS: 14.60.Pq; 13.10.+q; 25.30.Pt

1. Introduction

The atmospheric and solar neutrino oscillations observed in the Super-Kamiokande [1] and SNO [2] experiments have provided rather convincing evidence that neutrinos are massive and lepton flavors are mixed. To interpret the smallness of neutrino masses and the largeness of lepton flavor mixing angles, as indicated by current solar and atmospheric neutrino data, many theoretical models and phenomenological ansätze have been proposed [3]. Among them, the one proposed by Buchmüller and Wyler [4] is of particular interest, because (a) it is based on the simplest SO(10) lepton–quark mass relations [5] and the see-saw mechanism [6]; (b) it leads to quite specific predictions for the light neutrino masses, lepton flavor mixing angles, CP violation, and the neutrinoless double-β decay; and (c) it is able to predict the cosmological matter–anti-matter asymmetry via a very attractive mechanism—leptogenesis [7].

The present Letter aims to propose a non-SO(10) modification of the Buchmüller–Wyler ansatz, in order to account for current experimental data in a more accurate and more flexible way. To see why our effort makes sense, let us consider the following relation obtained by Buchmüller and Wyler [4]:

$$\epsilon^2 \approx \frac{(1+\tan^2\theta_{\rm atm})^3}{|\tan^2\theta_{\rm sun} - \cot^2\theta_{\rm sun}|} \frac{\Delta m^2_{\rm sun}}{\Delta m^2_{\rm atm}}, \tag{1}$$

where ϵ is a small expansion parameter of the Dirac neutrino mass matrix; $(\Delta m^2_{\rm sun}, \Delta m^2_{\rm atm})$ and $(\theta_{\rm sun}, \theta_{\rm atm})$ are the mass-squared differences and mixing angles of solar and atmospheric neutrino oscillations, respectively. From current Super-Kamiokande [1] and SNO [2] data, one obtains

$$\Delta m^2_{\rm sun} = (3.3\text{–}17) \times 10^{-5}\ {\rm eV}^2,$$

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PII: S0370-2693(02)02615-1

$$\tan^2\theta_{\rm sun} = 0.30\text{–}0.58 \tag{2}$$

at the 90% confidence level [8];[1] and

$$\Delta m^2_{\rm atm} = (1.6\text{–}3.9)\times 10^{-3}\ {\rm eV}^2,$$
$$\sin^2 2\theta_{\rm atm} > 0.92 \tag{3}$$

at the same confidence level [11]. Then it is straightforward to get $\epsilon > 0.1$ from Eqs. (1), (2) and (3). This lower bound of ϵ is not consistent with $\epsilon \approx \sqrt{m_u/m_c} \approx 0.04\text{–}0.08$ obtained from the SO(10) relation between Dirac neutrino and up-type quark mass matrices (i.e., $M_{\rm D} = M_{\rm u}$ [4]). Hence, one may wonder whether a modification of the Buchmüller–Wyler ansatz is possible, so as to avoid any inconsistency with data or any fine-tuning of the parameter space.

It is obvious that the Buchmüller–Wyler ansatz will get much flexibility to accommodate the present data on solar and atmospheric neutrino oscillations, if the relevant SO(10) lepton–quark mass relations are suspended. In this spirit, we propose a non-SO(10) modification of the Buchmüller–Wyler ansatz, in which charged lepton, Dirac and Majorana neutrino mass matrices have fewer free parameters. The predictability of such a new ansatz is expected to be more powerful. It is worthwhile to emphasize that we are following a purely phenomenological approach. We hope that the good agreement of our results with current experimental and observational data may shed light on some appropriate ways of model building, either within or beyond grand unified theories.

The remainder of this Letter is organized as follows. In Section 2, we specify the textures of charged lepton, Dirac and Majorana neutrino mass matrices, from which one may see both similarities and differences between the new ansatz and its original version. Section 3 is devoted to explicit predictions of this new ansatz for light neutrino masses, lepton flavor mixing angles, CP violation, neutrino oscillations, and the neutrinoless double-β decay. We calculate the lepton asymmetry in Section 4, and translate it into the baryon asymmetry via leptogenesis. Finally, a brief summary is given in Section 5.

[1] The best fit of current solar neutrino data in the large-angle Mikheyev–Smirnov–Wolfenstein (MSW) mechanism [9] yields $\Delta m^2_{\rm sun} = (5\text{–}7)\times 10^{-5}\ {\rm eV}^2$ and $\tan^2\theta_{\rm sun} = 0.35\text{–}0.45$ [10].

2. Lepton mass matrices

A simple extension of the standard model is to include one right-handed neutrino in each of three lepton families, while the Lagrangian of electroweak interactions keeps invariant under the ${\rm SU(2)_L}\times{\rm U(1)_Y}$ gauge transformation [7]. After spontaneous symmetry breaking, the lepton mass term can be written as

$$-\mathcal{L}_{\rm m} = \overline{(e\ \mu\ \tau)_{\rm L}} M_l \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\rm R} + \overline{(\nu_e\ \nu_\mu\ \nu_\tau)_{\rm L}} M_{\rm D} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm R} + \frac{1}{2}\overline{\left(\nu^{\rm c}_e\ \nu^{\rm c}_\mu\ \nu^{\rm c}_\tau\right)_{\rm L}} M_{\rm R} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm R} + {\rm h.c.}, \tag{4}$$

where $\nu^{\rm c}_\alpha \equiv C\bar{\nu}^{\rm T}_\alpha$ with C being the charge-conjugation operator (for $\alpha = e, \mu, \tau$); and M_l, $M_{\rm D}$ and $M_{\rm R}$ stand, respectively, for the charged lepton, Dirac neutrino and Majorana neutrino mass matrices. We expect that the scale of M_l and $M_{\rm D}$ is characterized by the gauge symmetry breaking scale $v \approx 175$ GeV. The scale of $M_{\rm R}$ may be much higher than v, because right-handed neutrinos are ${\rm SU(2)_L}$ singlets and their mass term is not subject to the electroweak symmetry breaking. As a consequence, the 3×3 light neutrino mass matrix M_ν arises from diagonalizing the 6×6 neutrino mass matrix

$$\mathbf{M}_\nu = \begin{pmatrix} 0 & M_{\rm D} \\ M^{\rm T}_{\rm D} & M_{\rm R} \end{pmatrix}, \tag{5}$$

and takes the see-saw form [6]

$$M_\nu \approx -M_{\rm D} M^{-1}_{\rm R} M^{\rm T}_{\rm D}. \tag{6}$$

Given specific textures of $M_{\rm D}$ and $M_{\rm R}$, one can calculate the mass eigenvalues of M_ν. The phenomenon of lepton flavor mixing at low energy scales stems from a non-trivial mismatch between diagonalizations of M_l and M_ν. In contrast, the lepton asymmetry at high energy scales depends on complex $M_{\rm D}$ and $M_{\rm R}$ [7].

Now let us propose a non-SO(10) modification of the Buchmüller–Wyler ansatz [4] for neutrino mixing and leptogenesis. First of all, we assume M_l and $M_{\rm D}$ to be symmetric matrices, just like $M_{\rm R}$. Second, we assume that the (1,1), (1,3) and (3,1) elements of M_l, $M_{\rm D}$ and $M_{\rm R}$ are all vanishing in a specific flavor basis, in analogy to a phenomenologically-favored texture of quark mass matrices $M_{\rm u}$ and $M_{\rm d}$ [12]. Note that we do not invoke any direct relationship between $(M_{\rm D}, M_l)$

and (M_u, M_d) (such as $M_D = M_u$ and $M_l = M_d$ in the SO(10) grand unified theory [4]). Instead, we assume that the non-zero elements of M_D and M_l can be expanded in terms of the Wolfenstein parameter $\lambda \approx 0.22$ [13]. It is well known that the mass spectra of charged leptons and quarks are hierarchical [14]:

$$\frac{m_e}{m_\tau} \sim \lambda^6, \qquad \frac{m_\mu}{m_\tau} \sim \lambda^2;$$
$$\frac{m_u}{m_t} \sim \lambda^8, \qquad \frac{m_c}{m_t} \sim \lambda^4;$$
$$\frac{m_d}{m_b} \sim \lambda^4, \qquad \frac{m_s}{m_b} \sim \lambda^2. \tag{7}$$

We conjecture that M_D might have a similar hierarchy as M_d, but its dominant mass eigenvalue should be close to the electroweak symmetry breaking scale $v \approx 175$ GeV. To be more explicit, we take

$$M_D = m_0 \begin{pmatrix} 0 & \hat{\lambda}^3 & 0 \\ \hat{\lambda}^3 & x\hat{\lambda}^2 & \hat{\lambda}^2 \\ 0 & \hat{\lambda}^2 & e^{i\zeta} \end{pmatrix},$$
$$M_l = m_\tau \begin{pmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & y\lambda^2 & \lambda^3 \\ 0 & \lambda^3 & 1 \end{pmatrix}, \tag{8}$$

where $\hat{\lambda} \equiv \lambda e^{i\omega}$; (x, y) are real and positive coefficients of $\mathcal{O}(1)$; and $m_0 \approx v$ holds. It is easy to check that three mass eigenvalues of M_D have the hierarchy $\lambda^4 : \lambda^2 : 1$, and those of M_l have the hierarchy shown in Eq. (7). The hierarchical structure of M_l implies that its contribution to lepton flavor mixing is very small and even negligible. Thus we expect that large lepton mixing angles observed in solar and atmospheric neutrino oscillations are essentially attributed to the light neutrino mass matrix M_ν in our ansatz.

Because the (1,1), (1,3) and (3,1) elements of both M_R and M_D have been assumed to be vanishing, M_ν must have the same texture zeros via the see-saw relation in Eq. (6) [15]. To generate a sufficiently large mixing angle in the ν_μ–ν_τ sector to fit current Super-Kamiokande data on atmospheric neutrino oscillations [1], the (2,2), (2,3), (3,2) and (3,3) elements of M_ν should be comparable in magnitude. This requirement is actually strong enough to constrain the texture of M_R in a quite unique way, as first observed by Buchmüller and Wyler [4]. For our purpose, we obtain[2]

$$M_R = M_0 \begin{pmatrix} 0 & \lambda^5 & 0 \\ \lambda^5 & z\lambda^4 & \lambda^4 \\ 0 & \lambda^4 & 1 \end{pmatrix}, \tag{9}$$

where z is a real and positive coefficient of $\mathcal{O}(1)$, and $M_0 \gg v$ holds. The texture of M_ν turns out to be

$$M_\nu = \frac{m_0^2}{M_0} \begin{pmatrix} 0 & \hat{\lambda} & 0 \\ \hat{\lambda} & z' & 1 \\ 0 & 1 & e^{i2\varphi} \end{pmatrix}, \tag{10}$$

where $z' \equiv 2x - ze^{i\omega}$ with $|z'| \sim \mathcal{O}(1)$, and $2\varphi \equiv 2\zeta - 5\omega$. Note that an overall phase factor $e^{i(5\omega-\pi)}$ has been omitted from the right-hand side of Eq. (10), since it has no contribution to lepton flavor mixing and CP violation at low energy scales.

We remark that the (2,2), (2,3), (3,2) and (3,3) elements of M_ν in Eq. (10) are all of $\mathcal{O}(1)$, from which a large mixing angle (around $\pi/4$) can be obtained for the ν_μ–ν_τ sector. It is due to such a prerequisite that the hierarchy of M_R can almost uniquely be fixed through the see-saw relation between M_R and M_ν. In other words, we essentially require little information about the ν_e–ν_μ sector of M_ν to arrive at Eq. (9).[3] As the ν_μ–ν_τ sector of M_ν is relatively insensitive to the renormalization effects from one scale to another [16], we expect that our phenomenological constraints on the texture of M_R at high energy scales make sense. To generate a large mixing angle in the ν_e–ν_μ sector to fit current Super-Kamiokande [1] and SNO [2] data on solar neutrino oscillations, the condition

$$\left| z' e^{i2\varphi} - 1 \right| \equiv \delta \sim \mathcal{O}(\lambda) \tag{11}$$

must be satisfied [4,17]. Some instructive constraints on the parameter space of x, z, ω and ζ can be drawn from Eq. (11), as one will see below.

It is worthwhile at this point to comment on two major differences of the present ansatz from its original version [4]:

[2] Note that M_R is given in terms of λ rather than $\hat{\lambda}$. If both M_D and M_R were expanded in terms of $\hat{\lambda}$, the resultant texture of M_ν would be unable to generate a large mixing angle in the ν_e–ν_μ sector.

[3] The ν_e–ν_μ sector of M_ν is generally sensitive to the renormalization effects, in particular when the corresponding mass eigenvalues (m_1 and m_2) are nearly degenerate [16].

(a) We do not assume any SO(10) lepton–quark mass relations. Therefore, the pattern of $M_{\rm D}$ can be taken as Eq. (8) with a structural hierarchy weaker than before. This modification will lead to a ratio of $\Delta m^2_{\rm sun}$ to $\Delta m^2_{\rm atm}$ at the percent level (i.e., ϵ^2 in Eq. (1) is replaced by λ^2), consistent very well with current experimental data. In this sense, we would say that the phenomenological success of this new ansatz may compensate for the theoretical cost for having discarded the simplest SO(10) mass relations.

(b) The number of free parameters in $M_{\rm D}$ and $M_{\rm R}$ is reduced from ten [4] to five (x, z, M_0, ω, and ζ). To do so, we have expanded $M_{\rm D}$ in terms of the complex parameter $\hat{\lambda}$ and $M_{\rm R}$ in terms of the real parameter λ. Such a treatment is plausible, since the Majorana neutrino mass matrix $M_{\rm R}$ is a priori independent of the Dirac neutrino mass matrix $M_{\rm D}$. While $|\hat{\lambda}| = \lambda$ holds by definition, it actually reflects the requirement of a large mixing angle in the ν_μ–ν_τ sector of M_ν, which imposes strict constraints on $M_{\rm D}$ and $M_{\rm R}$ via the see-saw mechanism. Because of the reduction of free parameters, the new ansatz is expected to have more powerful predictability.

3. Neutrino mixing

The symmetric neutrino mass matrix M_ν in Eq. (10) can be diagonalized by a 3×3 unitary matrix V,

$$V^\dagger M_\nu V^* = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \qquad (12)$$

where m_1, m_2 and m_3 are *physical* (real and positive) masses of three light neutrinos. As pointed out above, the contribution of M_l to lepton flavor mixing is expected to be very small and even negligible. Therefore the matrix V, which links the neutrino mass eigenstates (ν_1, ν_2, ν_3) to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$, can well describe the dominant effects of lepton flavor mixing at low energy scales. Current experimental data on solar, atmospheric and reactor neutrino oscillations [1,2,18] strongly suggest that $|V_{e3}| \ll 1$, $|V_{e1}| \sim |V_{e2}|$ and $|V_{\mu 3}| \sim |V_{\tau 3}|$ hold. Then a parametrization of V needs two big mixing angles (θ_x and θ_y) and one small mixing angle (θ_z) [19], in addition to a few complex phases. After some lengthy but straightforward calculations, we obtain[4]

$$V \approx \begin{pmatrix} c_x e^{i(\alpha-\gamma+\pi/2)} & s_x e^{i(\alpha-\gamma)} & s_z e^{i\alpha} \\ -s_x c_y e^{i(\beta+\gamma+\pi/2)} & c_x c_y e^{i(\beta+\gamma)} & s_y e^{i\beta} \\ s_x s_y e^{i(\zeta+\gamma+\pi/2)} & -c_x s_y e^{i(\zeta+\gamma)} & c_y e^{i\zeta} \end{pmatrix}, \qquad (13)$$

in which $s_a \equiv \sin\theta_a$ and $c_a \equiv \cos\theta_a$ (for $a = x, y, z$), $\alpha \equiv \omega + \zeta$, $\beta \equiv 5\omega - \zeta = \zeta - 2\varphi$, and $2\gamma \equiv \arg(z' e^{i2\varphi} - 1)$. The explicit expressions of three mixing angles $(\theta_x, \theta_y, \theta_z)$ are

$$\theta_x \approx \frac{1}{2}\arctan\left(2\sqrt{2}\,\frac{\lambda}{\delta}\right),$$
$$\theta_y \approx \frac{1}{2}\arctan\left(\frac{2}{\delta}\right),$$
$$\theta_z \approx \frac{1}{2}\arctan\left(\frac{\lambda}{\sqrt{2}}\right). \qquad (14)$$

In addition, three neutrino masses are given by

$$m_1 \approx \left(\frac{\lambda}{2\sqrt{2}}\tan\theta_x\right) m_3,$$
$$m_2 \approx \left(\frac{\lambda}{2\sqrt{2}}\cot\theta_x\right) m_3,$$
$$m_3 \approx 2\frac{m_0^2}{M_0}. \qquad (15)$$

We can see that θ_z is as small as we have expected, and a normal neutrino mass hierarchy $m_1 : m_2 : m_3 \sim \lambda : \lambda : 1$ shows up.

It is worth mentioning that our instructive results for (m_1, m_2, m_3) and $(\theta_x, \theta_y, \theta_z)$ will not get dramatic variations, if arbitrary coefficients of $\mathcal{O}(1)$ are taken for those non-zero elements in $M_{\rm D}$ and $M_{\rm R}$. The reason is simply that the hierarchical structures of $M_{\rm D}$ and $M_{\rm R}$ guarantee a stable texture of M_ν, from which the light neutrino masses and flavor mixing angles can straightforwardly be derived. For instance, a replacement $\lambda^4 \Rightarrow A\lambda^4$ with $|A| \sim \mathcal{O}(1)$ for the (2,3) and (3,2) elements of $M_{\rm R}$ does not affect the pattern of M_ν in the leading-order approximation [4]. Given a replacement $\hat{\lambda}^2 \Rightarrow B\hat{\lambda}^2$ with $|B| \sim \mathcal{O}(1)$ for the (2,3) and (3,2) elements of $M_{\rm D}$, the only variation of M_ν is that its corresponding (2,3) and (3,2) elements change

[4] Note again that an overall phase factor $e^{i(\pi-5\omega)/2}$ has been omitted from the right-hand side of Eq. (13), in accord with Eq. (10).

from 1 to B. In this case, we find

$$\theta_x \approx \frac{1}{2}\arctan\left[2\sqrt{1+|B|^2}\,\frac{\lambda}{\delta}\right],$$
$$\theta_y \approx \frac{1}{2}\arctan\left[\frac{2|B|}{1-|B|^2+\delta}\right],$$
$$\theta_z \approx \frac{1}{2}\arctan\left[\frac{2|B|\lambda}{(1+|B|^2)^{3/2}}\right] \tag{16}$$

and

$$m_1 \approx \left[\frac{\lambda}{(1+|B|^2)^{3/2}}\tan\theta_x\right]m_3,$$
$$m_2 \approx \left[\frac{\lambda}{(1+|B|^2)^{3/2}}\cot\theta_x\right]m_3,$$
$$m_3 \approx \left(1+|B|^2\right)\frac{m_0^2}{M_0}. \tag{17}$$

It is obvious that Eqs. (14) and (15) can be reproduced, respectively, from Eqs. (16) and (17) with the choice $|B| = 1$. Therefore, small deviations of $|B|$ from unity do not give rise to significant changes of the results obtained in Eqs. (14) and (15). Note that an arbitrary coefficient of $\mathcal{O}(1)$ for the (3,3) element of $M_{\rm D}$ or $M_{\rm R}$ can always be absorbed through a redefinition of the mass scale m_0 or M_0. On the other hand, an arbitrary coefficient of $\mathcal{O}(1)$ for the (1,2) and (2,1) elements of $M_{\rm D}$ or $M_{\rm R}$ can also be absorbed via a redefinition of the perturbative parameter $\hat{\lambda}$ or λ.

Some interesting implications of the simple results in Eqs. (14) and (15) are discussed in order.

(1) The hierarchy of three light neutrino masses allows us to determine the absolute value of m_3 from the observed mass-squared difference of atmospheric neutrino oscillations $\Delta m^2_{\rm atm} \equiv |m_3^2 - m_2^2| \approx m_3^2$. Using the recent Super-Kamiokande data listed in Eq. (2), we obtain

$$m_3 \approx \sqrt{\Delta m^2_{\rm atm}} \approx (4.0\text{–}6.2)\times 10^{-2}\ \text{eV}. \tag{18}$$

Given $m_0 \approx v$ for the Dirac neutrino mass matrix $M_{\rm D}$, the mass scale of three heavy Majorana neutrinos turns out to be

$$M_0 \approx 2\frac{v^2}{m_3} \approx (4.9\text{–}7.6)\times 10^{14}\ \text{GeV}. \tag{19}$$

We observe that this mass scale is not far away from the scale of grand unified theories $\Lambda_{\rm GUT} \sim 10^{16}$ GeV.

(2) The small parameter δ defined in Eq. (11) can well be constrained, if we take account of current experimental data on the mass-squared difference of solar neutrino oscillations $\Delta m^2_{\rm sun} \equiv |m_2^2 - m_1^2|$ shown in Eq. (2). As the ratio $R \equiv \Delta m^2_{\rm sun}/\Delta m^2_{\rm atm}$ is given by

$$R \approx \frac{\delta}{16}\sqrt{8\lambda^2+\delta^2} \approx (0.85\text{–}10.6)\times 10^{-2}, \tag{20}$$

we obtain $\delta \approx 0.21$–1.2 for $\lambda \approx 0.22$. Note that $\delta > 0.5$ is apparently in conflict with our original assumption $\delta \sim \mathcal{O}(\lambda)$ in Eq. (11). Therefore, the reasonable range of δ should be $\delta \approx 0.21$–0.50, which leads in turn to $R \approx (0.85\text{–}2.5)\times 10^{-2}$. Subsequently we fix $\delta = \sqrt{2}\,\lambda \approx 0.31$ as a typical input.

(3) Using $\delta = \sqrt{2}\,\lambda$, we explicitly obtain

$$\theta_x \approx 31.7^\circ, \qquad \theta_y \approx 40.6^\circ, \qquad \theta_z \approx 4.4^\circ. \tag{21}$$

To a good degree of accuracy, the mixing factors of solar, atmospheric and reactor neutrino oscillations are associated, respectively, with θ_x, θ_y and θ_z [20]. From Eq. (14) or Eq. (21), we get

$$\sin^2 2\theta_{\rm sun} \approx \sin^2 2\theta_x \approx \frac{8\lambda^2}{8\lambda^2+\delta^2} \approx 0.8,$$
$$\sin^2 2\theta_{\rm atm} \approx \sin^2 2\theta_y \approx \frac{4}{4+\delta^2} \approx 0.98,$$
$$\sin^2 2\theta_{\rm rea} \approx \sin^2 2\theta_z \approx \frac{\lambda^2}{2} \approx 0.024. \tag{22}$$

Note that we have kept the δ-induced correction to $\sin^2 2\theta_{\rm atm}$, in order to illustrate its small departure from unity (maximal mixing). The typical results in Eq. (22) are in good agreement with current Super-Kamiokande [1], SNO [2] and CHOOZ [18] data.

(4) Due to the mass hierarchy of three light neutrinos, our ansatz predicts a relatively small value for the effective mass term of the neutrinoless double-β decay:

$$\langle m\rangle_{ee} \equiv \sum_{k=1}^{3}\left(m_k V_{ek}^2\right) \approx \frac{\lambda^2}{8}m_3$$
$$\approx (2.4\text{–}3.8)\times 10^{-4}\ \text{eV}, \tag{23}$$

which seems hopeless to be detected in practice. Indeed the present experimental upper bound is $\langle m\rangle_{ee} < 0.35$ eV at the 90% confidence level [21].

(5) CP or T violation in normal neutrino oscillations is measured by a universal and rephrasing-invariant parameter $\mathcal{J}$ [22], which can be calculated

as follows:

$$\mathcal{J} = \left|\operatorname{Im}\left(V_{e2}V_{\mu3}V_{e3}^*V_{\mu2}^*\right)\right| \approx \frac{\lambda^2 \sin 2\gamma}{4\sqrt{8\lambda^2+\delta^2}}, \tag{24}$$

where

$$\sin 2\gamma \approx \frac{2x}{\delta}\sin 2\varphi - \frac{z}{\delta}\sin(2\varphi+\omega). \tag{25}$$

Because of $x \sim z \gg \delta \sim \lambda$, a significant cancellation on the right-hand side of Eq. (25) is naturally expected. There exists an interesting parameter space, in which

$$x = \frac{1}{\sqrt{2}}, \qquad z = 1+\sqrt{2}\,\lambda,$$
$$\zeta = -\omega = \frac{\pi}{4}. \tag{26}$$

Considering Eq. (11), one may easily check that $\delta = \sqrt{2}\,\lambda$ *does* hold for the chosen values of x, z, ω and ζ. It is particularly amazing that $\sin 2\gamma = 1$ holds in this case. Therefore, we obtain $\mathcal{J} \approx \lambda/(4\sqrt{10}) \approx 2\%$. CP violation at the percent level could be measured in the future at neutrino factories [23].

4. Leptogenesis

The symmetric neutrino mass matrix $M_{\rm R}$ in Eq. (9) can be diagonalized by a 3×3 unitary matrix U,

$$U^\dagger M_{\rm R} U^* = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \tag{27}$$

where M_1, M_2 and M_3 are *physical* (real and positive) masses of three heavy Majorana neutrinos. In the leading-order approximation, we obtain

$$M_1 \approx \frac{\lambda^6}{z}M_0,$$
$$M_2 \approx z\lambda^4 M_0,$$
$$M_3 \approx M_0, \tag{28}$$

and

$$U \approx \begin{pmatrix} i & \dfrac{\lambda}{z} & 0 \\ -i\dfrac{\lambda}{z} & 1 & \lambda^4 \\ i\dfrac{\lambda^5}{z} & -\lambda^4 & 1 \end{pmatrix}. \tag{29}$$

One can see that the masses of three heavy Majorana neutrinos perform a clear hierarchy. In view of Eq. (19), we arrive explicitly at

$$\{M_1, M_2, M_3\} \approx \left\{5.2\times10^{10},\ 1.8\times10^{12},\ 6.0\times10^{14}\right\}\ \text{GeV}, \tag{30}$$

if $M_0 = 6.0\times10^{14}$ GeV and $z = 1+\sqrt{2}\,\lambda$ are typically taken.

A lepton asymmetry may result from the interference between tree-level and one-loop amplitudes of the decay of the *lightest* heavy Majorana neutrino with mass M_1 [7]. This asymmetry can be expressed, in the physical basis where $M_{\rm R}$ is diagonal and the Dirac neutrino mass matrix takes the form $M_{\rm D}U^*$ instead of $M_{\rm D}$, as [24]

$$\varepsilon_1 \approx -\frac{3}{16\pi v^2}\frac{M_1}{[U^{\rm T}M_{\rm D}^\dagger M_{\rm D}U^*]_{11}} \times \sum_{j=2}^{3}\frac{\operatorname{Im}([U^{\rm T}M_{\rm D}^\dagger M_{\rm D}U^*]_{1j})^2}{M_j}, \tag{31}$$

where $v \approx 175$ GeV denotes the electroweak scale. In writing out Eq. (31), we have taken account of the strong mass hierarchy $M_1 \ll M_2 \ll M_3$. With the help of Eqs. (8), (28) and (29), we get

$$\varepsilon_1 \approx -\frac{3\lambda^6}{16\pi} \times \frac{x^2 z\sin 2\omega - 2x(1+x^2)\sin\omega + \sin 2(2\omega-\zeta)}{z(1+x^2+z^2-2xz\cos\omega)}. \tag{32}$$

Once the parameters x, z, ω and ζ are specified, one will be able to predict the magnitude of ε_1 from Eq. (32).

For the purpose of illustration, we adopt the specific parameter space given in Eq. (26) to evaluate the size of ε_1. The result is

$$\varepsilon_1 \approx -\frac{\lambda^6}{4\pi}\left(1 - \frac{23\sqrt{2}}{12}\lambda + \frac{67}{18}\lambda^2\right), \tag{33}$$

or numerically $\varepsilon_1 \approx -5.2\times10^{-6}$. To translate this lepton asymmetry into the baryon asymmetry of the universe [7], one needs to calculate a suppression factor κ induced by the lepton-number-violating wash-out processes [25]. Note that κ depends closely on the following quantity:

$$K_{\rm R} \equiv \frac{[U^{\rm T} M_{\rm D}^{\dagger} M_{\rm D} U^*]_{11}}{8\pi v^2} \frac{M_{\rm Pl}}{1.66\sqrt{g_*}\, M_1} \approx \frac{3-\sqrt{2}\lambda + 6\lambda^2}{16\pi} \frac{M_{\rm Pl}}{1.66\sqrt{g_*}\, M_0}, \tag{34}$$

which characterizes the out-of-equilibrium decay rate of the *lightest* heavy Majorana neutrino with mass M_1. In Eq. (34), $g_* \approx 100$ represents the number of massless degree of freedom at the time of the decay, and $M_{\rm Pl} \approx 1.22 \times 10^{19}$ GeV is the Planck mass scale. Making use of the typical inputs taken above for Eq. (30), we arrive at $K_{\rm R} \approx 73$. The suppression factor κ can then be calculated with the help of an approximate parametrization [25] obtained from integrating the Boltzmann equations (for $10 \leqslant K_{\rm R} \leqslant 10^6$):

$$\kappa \approx \frac{0.3}{K_{\rm R}} \frac{1}{(\ln K_{\rm R})^{0.6}} \approx 1.7 \times 10^{-3}. \tag{35}$$

Finally we get an instructive prediction for the asymmetry between baryon ($n_{\rm B}$) and anti-baryon ($n_{\overline{\rm B}}$) numbers of the universe:

$$Y_{\rm B} \equiv \frac{n_{\rm B} - n_{\overline{\rm B}}}{\mathbf{s}} = \frac{c\kappa\varepsilon_1}{g_*} \approx 4.7 \times 10^{-11}, \tag{36}$$

where $\mathbf{s}$ denotes the entropy density, and $c = -8/15$ describes the fraction of ε_1 converted into $Y_{\rm B}$ via sphaleron processes in the framework of three lepton–quark families and two Higgs doublets [26]. One can see that our result is consistent quite well with the observed baryon asymmetry, $Y_{\rm B} \approx (1\text{–}10) \times 10^{-11}$ [27].

Of course, one may go beyond the typical parameter space taken in Eq. (26) to make a delicate analysis of all measurables or observables, only if the condition in Eq. (11) is satisfied. It is remarkable that we can quantitatively interpret both the baryon asymmetry of the universe and the small mass-squared differences and large mixing factors of solar and atmospheric neutrino oscillations. In this sense, our ansatz is a *complete* phenomenological ansatz favored by current experimental and observational data, although it has not been incorporated into a convincing theoretical model. A number of different ansätze on leptogenesis and neutrino oscillations have recently been proposed [28,29], but some of them turn to be ruled out by the present Super-Kamiokande and SNO data.

Future neutrino experiments will test the present ansatz and help to distinguish it from other viable models in the following four aspects:

(a) Our ansatz predicts the ratio of $\Delta m^2_{\rm sun}$ to $\Delta m^2_{\rm atm}$ (i.e., R) to be around 1%. If more accurate solar and atmospheric neutrino data yield $R < 0.5\%$ or $R > 5\%$, our ansatz will somehow become disfavored.

(b) The prediction of our ansatz for the smallest lepton mixing angle is quite certain: $\theta_z \approx \lambda/(2\sqrt{2})$ (or 4.4°). This result may easily be examined in a variety of long-baseline neutrino oscillation experiments [30]. Some other ansätze [28] have given quite different predictions for θ_z, either much larger or much smaller than ours.

(c) The magnitude of $\langle m\rangle_{ee}$ predicted by our ansatz (of order 10^{-4} eV) is too small to be measured in any proposed experiments for the neutrinoless double-β decay [31]. If an unambiguous signal of the neutrinoless double-β decay is observed in the near future, the present ansatz will definitely be ruled out.

(d) In our ansatz, the rephrasing-invariant parameter of CP violation (i.e., $\mathcal{J}$) is predicted to be at the percent level. This strength of leptonic CP or T violation could be detected in the far future at neutrino factories [23]. Therefore, another criterion to discriminate between our ansatz and other viable models is to see how large the CP-violating effects can be in neutrino oscillations.

5. Summary

In summary, we have proposed a non-SO(10) modification of the Buchmüller–Wyler ansatz for neutrino mixing and leptogenesis. Its consequences on the light neutrino masses, lepton flavor mixing angles, the neutrinoless double-β decay, and the baryon asymmetry are all in good agreement with current experimental and observational data. In particular, an indirect connection between the lepton asymmetry at high energy scales and CP violation in neutrino oscillations has shown up in such a specific ansatz. We expect that phenomenological ansätze of this nature will get more stringent tests in the era of long-baseline neutrino oscillation experiments. On the theoretical side, a deeper understanding of the origin of fermion masses, flavor mixing and CP violation becomes more desirable than before.

Acknowledgements

The author would like to thank H. Fritzsch for warm hospitality in Universität München, where part of this work was done. He is also grateful to W. Buchmüller for enlightening discussions in Bamberg, and to H. Fritzsch for helpful comments in Amsterdam.

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